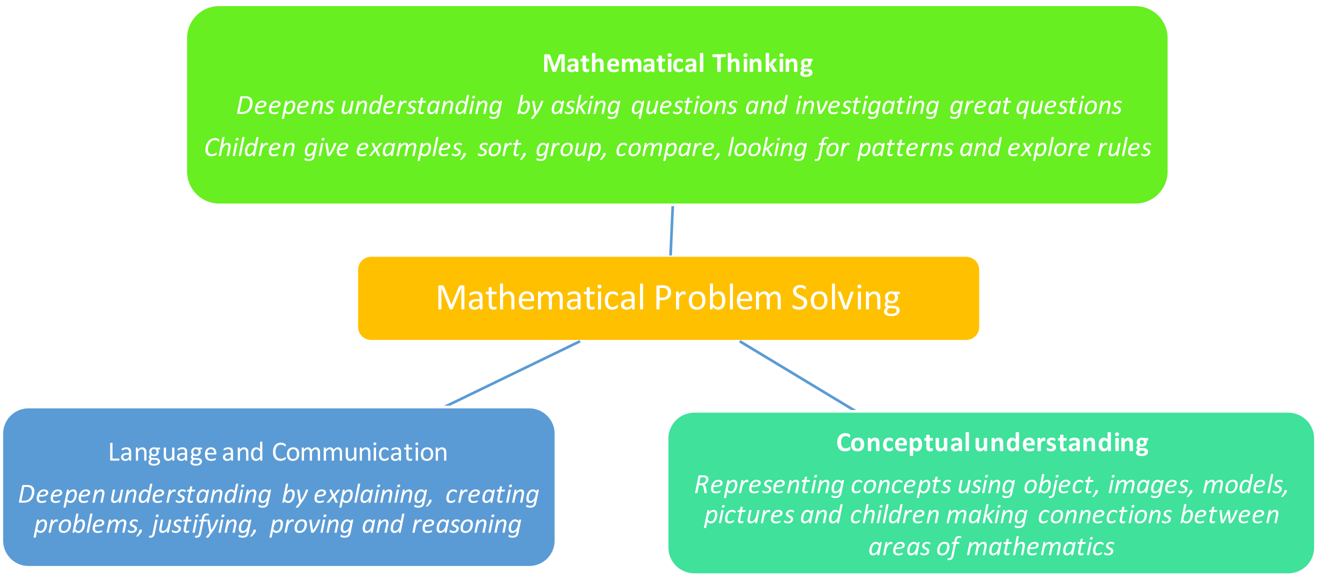
Thinking Apples – explaining reasoning skills for problem solving in Mathematics

Developing reasoning skills that facilitate problem solving mathematics can be challenging. To be an effective problem solver in Mathematics, learners must be equipped with the metacognitive skills to recognise what knowledge and skills they may need to solve a problem. The EEF outlines 7 recommendations to develop learners' metacognitive skills:

- Teachers need to understand how to develop metacognitive skills and how to make these skills explicit to children (i.e. awareness of strengths and weaknesses, understanding how they learn best)
- Explicitly teaching metacognitive strategies and methods to plan, monitor and evaluate their learning (i.e. 'I'm using trial and error but it's taking a long time. I'll reread the question and consider whether I could look at the problem a different way – **switching from trial and error to conjecture**)
- Model own thinking
- Set appropriate level of challenge (i.e. pitch the content of a task lower to help teach a strategy – **work systematically to find all the multiples of 2 between 0 and 10**)
- Promote and develop metacognitive talk
- Explicitly teach children how to organise and effectively manage their learning
- Schools offer CPD to develop teacher knowledge of metacognition

Metacognition and Self-regulated Learning, EEF (2018)
(https://educationendowmentfoundation.org.uk/tools/guidance-reports/metacognition-and-self-regulated-learning/)

CPD on the pedagogical skills required to develop the metacognitive skills and reasoning skills outlined in this document will be delivered. For clarity, this document will focus on the reasoning skills specific to Mathematics that the Thinking Apples aims to support



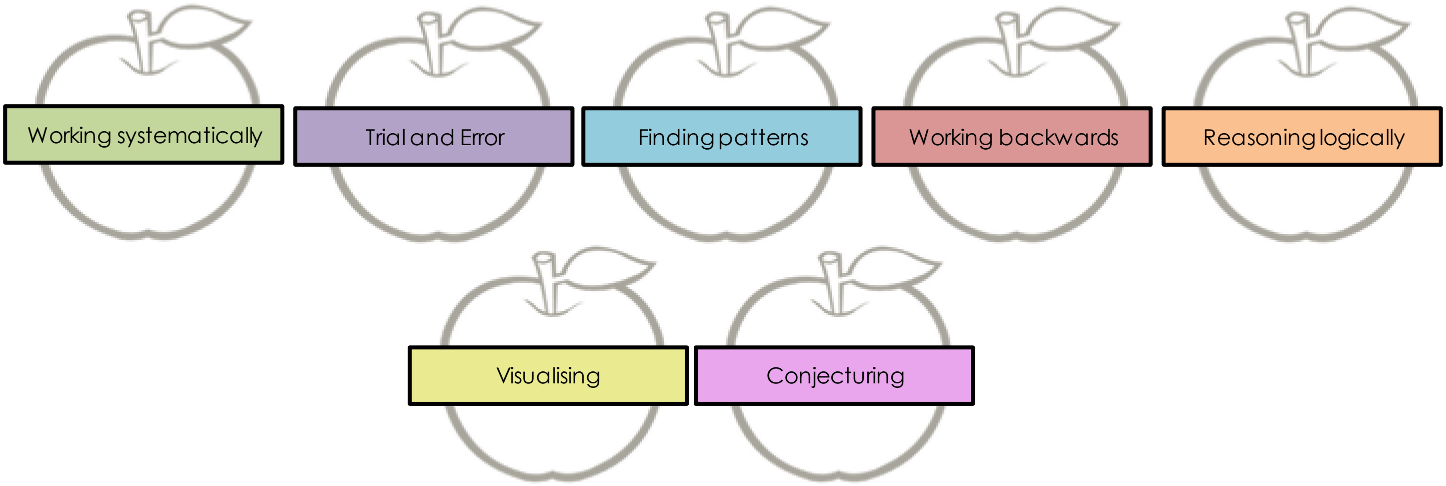
Helen Drury's model (2018) highlights three key areas to develop mathematical problem solving (shown above). Our curriculum facilitates language and communication and through the concrete – pictorial – abstract approach, it also nurtures a conceptual understanding of mathematics. The Thinking Apples look to develop mathematical thinking as an explicit skill to apply to a variety of problem-solving contexts within Mathematics.

We aim to equip our children with the reasoning skills required to problem solve through three ways:

- Choice of task
- Adaptive teaching and modelling of the problem-solving process
- Explicit and frequent opportunities for our learners to apply reasoning skills to a problem-solving context.

(NRICH, 2014)

Firstly, reasoning skills required to problem solve is promoted through our choice of task. Teachers pick reasoning and problems solving tasks that lend themselves to a focus upon a particular skill. Secondly, through the modelling process of our lessons and 'Think aloud' and collaborative strategies to support our children's development of mathematical thinking. Finally, by offering explicit and frequent opportunities for children to utilise their reasoning skills to solve problems. To support our children in developing reasoning skills that can be transferred across mathematical problems, our developing learning within lessons focuses upon the 'Thinking Apples' to help make metacognitive processes explicit for our children.

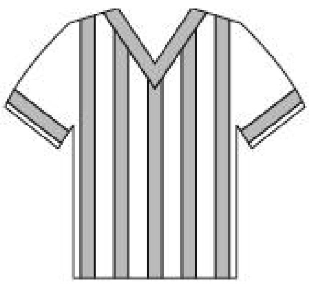


Not all skills will be applicable or useful to solve all problems and some problems may involve more than one skill or could be solved in more than one way. As much as possible, children should be encouraged to find different methods and celebrate different possibilities and, where applicable, multiple possible answers.

Working systematically

Adam chooses the colours for a new team shirt. The shirt has two colours
There are four colours to choose from: yellow, blue, white and red. Write the two missing combinations.
The shirt could be:

- *Yellow and blue*
- *Yellow and white*
- *Yellow and red*
- *Blue and white*



Working in an order that can clearly show others a pattern or system has been used. For example, these jottings show a systematic approach:

	Y	B	W	R
Y		Y + B	Y + W	Y + R
B	B + Y		B + W	B + R
W	W + Y	W + B		W + R
R	R + Y	R + B	W + R	

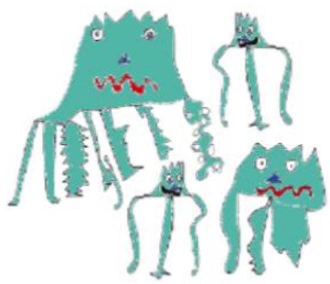
Working systematically through all the possible combinations highlights that only Blue and Red and White and Red have been left out.

Trial and Error

Zios and Zepts

Age 7 to 11 ★

On the planet Vuv there are two sorts of creatures. The Zios have 3 legs and the Zepts have 7 legs.



The great planetary explorer Nico, who first discovered the planet, saw a crowd of Zios and Zepts. He managed to see that there was more than one of each kind of creature before they saw him. Suddenly they all rolled over onto their backs and put their legs in the air.

He counted 52 legs. How many Zios and how many Zepts were there? Do you think there are any different answers?

You try out an answer. It either works or doesn't work, but you use the answer to decide whether you need to continue solving the problem or if you have found all the possibilities. Often trial and error works well with working systematically.

Finding patterns

Pattern spotting can make problem solving faster and by questioning why particular patterns occur, we can deepen our mathematical knowledge. Our Maths Meetings structure in Year 3 and 4 draws attention to this, developing reasoning knowledge around the relationships between the 3 and 6 timestables, for example, to consolidate and secure knowledge of key facts. Articulating why 2 x 3 is the same as 1 x 6 can help lay the foundations for understanding problems such as this one:

33,630 = 354 × 95

Use this multiplication to complete the calculations below.

354 × 9.5 =

3,540 × 95 =

3,363 ÷ 95 =

2 marks

Working backwards

Junior Frogs

Age 5 to 11 ★★

This challenge is based on the game Frogs which you may have seen before.

There are two blue frogs and two red frogs.

A frog can jump over one other frog onto an empty lilypad or it can slide onto an empty lilypad which is immediately next to it.
Only one frog, at a time, is allowed on each lilypad.

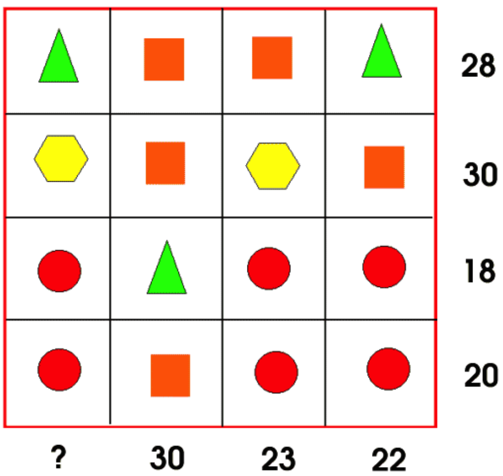
Now the idea is for the blue frogs and red frogs to change places. So, the red frogs will end up on the side where the blue frogs started and the blue frogs will end up where the red frogs began.

The challenge is to do this in as few slides and jumps as possible.

Staring at the end of a problem can enable learners to find the most efficient route and effectively eliminate strategies that would not work.

Reasoning logically

This skill requires learners to draw upon prior knowledge and explain how it applies logically to a new concept or statement. One example could be this activity.



Other examples include statement exercises such as 'Always, sometimes or never?'

Visualising

Visualising may enable learners to see solutions that may not be apparent when the problem is in word form. For example:

Odd Squares

Age 7 to 11 ★

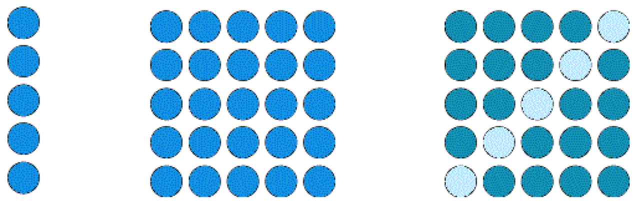
Think of a number.
Square it.
Subtract your starting number.

Is the number you're left with odd or even?

Try with other numbers.

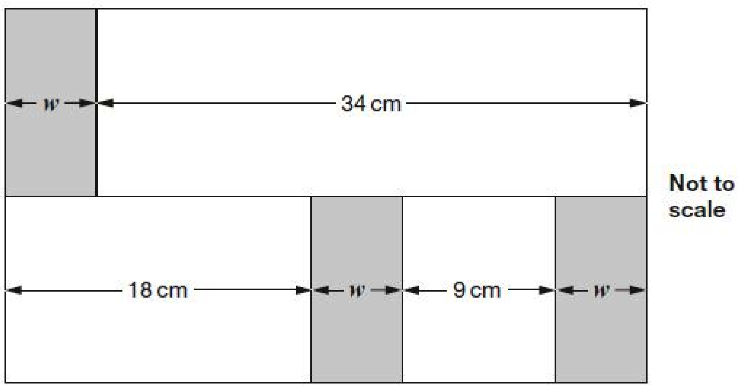
What do you notice?

How do these images help you explain your observations?



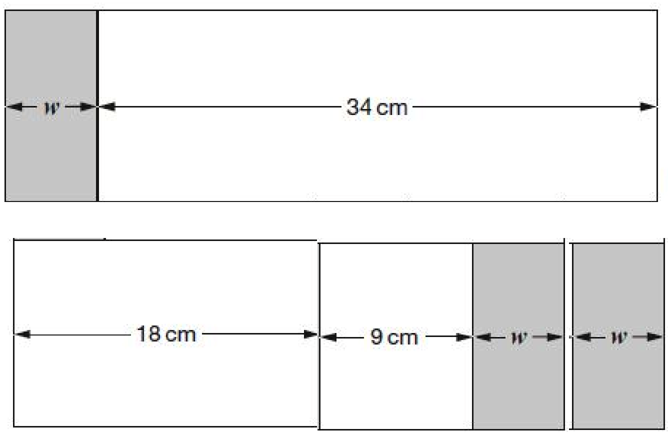
Another example might be this KS2 problem:

In this diagram, the shaded rectangles are all of equal width (*w*).



Calculate the width (*w*) of one shaded rectangle.

This could be redrawn like this to show the difference in values:



From the above we can see that 34 = 18 + 9 + w

Conjecturing

Conjecturing means that learners are beginning to make generalisations and links about their mathematical learning. For example, when presented with 'Always, sometimes, never? All even numbers end in 0, 2, 4, 6 and 8' might cause some children to question whether negative numbers or decimals can be considered even and provoke a deeper understanding of the mathematical concept.

Guidance adapted from:* *https://nrich.maths.org/11082